№ 3.2.1 $E = -\nabla\varphi$

$$\vec{E} = -\frac{a}{d}\cdot\frac{1}{\frac{a}{2} - z + \sqrt{\left(\frac{a}{2}-z\right)^2 + x^2+y^2}}\cdot\frac{1}{\frac{a}{2}+z-\sqrt{\left(\frac{a}{2}+z\right)^2 + x^2+y^2}}\cdot\frac{1}{-\frac{a}{2} - z + \sqrt{\left(\frac{a}{2}+z\right)^2+x^2+y^2}}\cdot\frac{x}{\sqrt{\left(\frac{a}{2}+z\right)^2+x^2+y^2}}$$

$$-\left(\frac{a}{a}\cdot\frac{\frac{a}{2}+z-\sqrt{\left(\frac{a}{2}+z\right)^2+x^2+y^2}}{\frac{a}{2}-z+\sqrt{\left(\frac{a}{2}+z\right)^2+x^2+y^2}}\cdot\left(\frac{\cdots}{\cdots}\right)+\frac{\partial\varphi}{\partial x}+\frac{\partial\varphi}{\partial z}\right.$$

$$\frac{\partial\varphi}{\partial y}$$

$r \to 0$: $E = \frac{a}{a}\cdot\frac{1}{a-2z}\cdot\infty \to \infty$

$r \to \infty$; $E \to \infty$

$$\Delta\varphi = \int_V \frac{\rho(r)\,dr}{|r - r'|}, \quad \rho\,dV = \sigma\,dS; \quad \Delta\varphi(r) = \int_S \frac{\sigma\,dS}{|r-r'|}$$

$$\varphi = 2h\int\frac{R\,dR}{|r - r'|}; \quad (r - r') = r^2 + R^2 - 2rR\sin\theta\cos\alpha$$

$x = r\sin\theta \qquad x' = r\cos\alpha'$

$y = 0 \qquad y' = R\sin\alpha'$

$z = r\cos\theta \qquad z' = 0$

$$\varphi = \frac{a}{(b^2-a^2)\pi}\cdot\frac{1}{2}(b^2-a^2)\int\frac{d\alpha'}{\sqrt{r^2+R^2-2rR\sin\theta\cos\alpha'}} =$$

$$= \left|\alpha' = \pi - 2\beta\right| = \frac{aR^2}{2(b^2-a^2)\pi}\int_{-\pi/2}^{\pi/2}\frac{d\beta}{\sqrt{r^2+R^2-2rR\sin\theta(2\sin\beta-1)}}$$

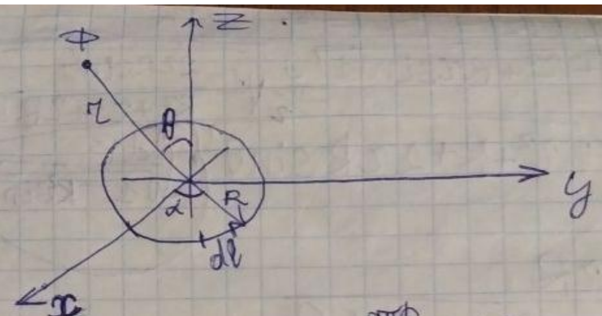
$$\varphi(r,\theta) = \ldots\cdot K \Rightarrow$$

$$K = \sqrt{\frac{4rR\sin\theta}{r^2+R^2+2rR\sin\theta}}$$

№ 3.2.2

q, R

$\Phi - ?$



$\Phi(\vec r) = \int \frac{\rho(\vec r')\,dV'}{|\vec r - \vec r'|}$; $\Phi(\vec r) = \int_0^{2\pi R} \frac{\lambda\, dl}{|\vec r - \vec r'|}$; $\lambda = \frac{q}{2\pi R}$

$\vec r(x,0,z) = (r\sin\theta, 0, r\cos\theta)$

$\vec r'(x',y',0) = (R\cos\alpha, R\sin\alpha, 0)$

$dl = R\,dl$

$|\vec r - \vec r'|^2 = (r\sin\theta - R\cos\alpha)^2 + R^2\sin^2\alpha + r^2\cos^2\theta =$

$= r^2\sin^2\theta - 2rR\sin\theta\cos\alpha + R^2\cos^2\alpha + R^2\sin^2\alpha +$

$+ r^2\cos^2\theta = r^2 + R^2 - 2rR\sin\theta\cos\alpha$

$\Phi(\vec r) = \lambda\int_0^{2\pi R}\frac{R\,dl}{\sqrt{r^2+R^2-2rR\sin\theta\cos\alpha}} = \frac{\lambda R}{\sqrt{2rR\sin\theta}}\int_0^{2\pi R}\frac{dl}{\sqrt{\frac{r^2+R-\cos\alpha}{2rR\sin\theta}}} =$

$= \left| \begin{matrix}\alpha = \pi - 2\beta \\ \beta = \frac{\pi-\alpha}{2}\end{matrix} \right| \begin{matrix}\cos(\pi-2\beta) = -\cos(2\beta)\\ dl = 2d\beta\end{matrix} \Bigg| \Rightarrow$

$\Phi(\vec r) = \frac{\lambda R\cdot 2}{\sqrt{2rR\sin\theta}}\int_{-\pi/2}^{\pi/2}\frac{d\beta}{\sqrt{\frac{r^2+R^2}{2rR\sin\theta}+\cos 2\beta}} = 2\lambda R\int\frac{d\beta}{\sqrt{R^2+r^2+rR\sin\theta(1-2\sin^2\beta)}}$ $\textcircled{=}$ $R^2+r^2+Rr\sin\theta-2rR\sin\theta\cdot$

$\cdot\sin^2\beta = (R^2+r^2+rR\sin\theta)\left(1-\frac{2rR\sin\theta}{R^2+r^2+rR\sin\theta}\cdot\sin^2\beta\right)$

$\textcircled{=}\ \frac{2\lambda R}{\sqrt{R^2+r^2+Rr\sin\theta}}\int_{-\pi/2}^{\pi/2}\frac{d\beta}{\sqrt{1-\frac{2rR\sin\theta}{R^2+r^2+rR\sin\theta}\sin^2\beta}} =$

$= \frac{2q}{\pi}\sqrt{r^2+R+2rR\sin\theta}\cdot\int_0^{2\pi}\frac{d\beta}{\sqrt{1-k\sin\beta}} = k$

$k^2 = \frac{4rR}{r^2+R^2+2rR\sin\theta}$